Scalar mesons in weak semileptonic decays of $B_{(s)}$

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The transition form factors of $B_{(s)} \to S$, with S denoting a scalar meson, are investigated in the light-cone sum rules approach. The numerical values are approximately twice as large as that estimated in the light-front quark model and QCD sum rules approach. Using these form factors, we present the analysis of the decay rates for $B \to a_0(1450)l\bar{\nu}_l$, $B \to K_0^*(1430)l\bar{l}$, $B_s \to K_0^*(1430)l\bar{\nu}_l$ and $B_s \to f_0(1500)l\bar{l}$ with $l = e, \mu, \tau$. The results indicate that magnitudes of $BR(\bar{B}_0 \to a_0(1450)l\bar{\nu}_l)$ and $BR(B_s \to K_0^*(1430)l\bar{\nu}_l)$ can arrive at the order of 10^{-4}, which can be measured in the future experiments to clarify the inner structure of scalar mesons. It is also observed that $BR(B \to K_0^*(1430)\tau^+\tau^-)$ and $BR(B_s \to f_0(1500)\tau^+\tau^-)$ are an order of magnitude smaller than the corresponding channels of e^+e^- and $\mu^+\mu^-$ final states due to the heavily suppressed phase space . Moreover, the longitudinal lepton polarization asymmetry for $B \to K_0^*(1430)l\bar{l}$ and $B_s \to f_0(1500)l\bar{l}$ are also investigated, whose values are close to -1 for the e^+e^- and $\mu^+\mu^-$ pair except the region close to the end points.

I. INTRODUCTION

The inner structure of scalar mesons has been controversial for over three decades, which makes them one of the alluring issues in contemporary particle physics. In particular, the existence of some physical states, such as $f_0(1370)$, is still in dispute due to the absence of convincing evidence [1]. It is suggested that the scalar mesons with the masses below and above 1GeV can be organized into two nonets in terms of their spectrum [2, 3]. The flavor singlet $f_0(600)$ (or σ), $f_0(980)$, the isodoublet $K_0^*(800)$ (or κ), and the isovector $a_0(980)$ constitute the nonet below 1 GeV; while $f_0(1370)$, $f_0(1500)/f_0(1710)$, $K_0^*(1430)$ and $a_0(1450)$ form the other one near 1.5 GeV [2, 3]. Up to now, there is no general agreement on the nature of these states [4] due to the ambiguity existing in all the available interpretations including conventional $q\bar{q}$ states [3], glueball, hybrid states, molecule states [5–7] as well as tetra-quark states [8] and the superpositions of these contents [9–11]. Among all the scalar mesons, the $K_0^*(1430)$ is predominantly viewed as $s\bar{u}$ or $s\bar{d}$ state in almost all the models. Hence, it is justified to assume that $a_0(1450)$, $K_0^*(1430)$ and $f_0(1370)$ being in the same nonent are respectively the $u\bar{d}$, $u\bar{s}$ and $u\bar{u} + d\bar{d}$ states, based on the naive quark model, which is also the picture of scalar mesons adopted in this paper.

Studies on the strong and electromagnetic decays of scalar decays have been received extensive interests in the literature [12–15]. Besides, the production properties of scalar mesons in πN scattering, $p\bar{p}$ annihilation, $\gamma\gamma$ formation

and heavy meson decays can also serve as an ideal platform to explore the underlying structures of scalar mesons as well as the non-perturbative dynamics of QCD. Thanks to the progress of accelerator and detector techniques, both Belle and BaBar have observed strong indications of scalar mesons within a broad spectrum between 1.0 and 1.5 GeV in B meson decays [16]. In this work, we will focus on the semi-leptonic weak production of scalars in $B_{(s)}$ decays, which are relatively clean compared to the hadronic decays from the theoretical viewpoint. Moreover, semileptonic decays of $B_{(s)}$ mesons are also of great importance to determine the quark-flavor mixing matrix — the Cabibbo-Kobayashi-Maskawa (CKM) matrix [17, 18] and testing its unitarity under the requirement of the standard model (SM).

The main job of investigating the semi-leptonic decays of $B_{(s)}$ to the scalar mesons (S) is to properly evaluate the hadronic matrix elements for $B_{(s)} \to S$, namely the transition form factors, which are governed by the non-perturbative QCD dynamics. Several methods exist in the literature to deal with this problem, such as simple quark model [19], light-front approach [20–22], QCD sum rules (QCDSR) [23, 24], light-cone QCD sum rules (LCSR) [25–27], perturbative QCD factorization approach [28–30]. The QCD sum rules approach, which is a fully relativistic approach and well rooted in quantum field theory, has made a tremendous success; however, short distance expansion fails in non-perturbative condensate when applying the three-point sum rules to the computations of form factors in the large momentum transfer or large mass limit of heavy meson decays. As a marriage of standard QCDSR technique [23, 24] and theory of hard exclusive process [31–38], LCSR cure the problem of QCDSR applying to the large momentum transfer by performing the operator product expansion(OPE) in terms of twist of the revelent operators rather than their dimension [39]. An important advantage of light-cone QCD sum rules is that it allows a systematic inclusion of both hard scattering effects and the soft contributions [40]. Phenomenologically, LCSR has been widely applied to investigate the semi-leptonic decays of heavy hadrons [41–44], radiative hadronic decays [45–47], non-leptonic two body decays of B meson [48–51] and strong coupling constants [52].

In the present work, we would like to adopt LCSR approach to study the rare decay of $B_{(s)} \to S$. The essential inputs in the light-cone QCD sum rules is the hadronic distribution amplitudes other than vacuum condensates in the QCD sum rules. It is known that LCDAs are non-perturbative functions, which describes the hadronic structure in rare parton configurations with a fixed number of Fock components at small transverse separation in the infinite momentum frame. In an attempt to accommodate the experimental data, there have been continuous interests concentrating on the research of pre-asymptotic corrections to the distribution amplitudes of hadrons in the exclusive reactions over two decades. In particular, the leading twist and twist-3 distribution amplitudes of scalar mesons have been worked out in [3, 53] based on the QCD sum rules and conformal symmetry hidden in the QCD Lagrangian and we will use these amplitudes in this paper.

The paper is organized as follows: In section II we present the effective Hamiltonian responsible for the $b \to u, s$ transitions in the standard model, where the parameterizations of hadronic matrix elements are also collected here. Based on the trace formulae and equation of motion, we also derive the relations among form factors $f_+(q^2)$, $f_-(q^2)$ and $f_T(q^2)$ in the large recoil and heavy quark limit. The Gegenauber moments of twist-2 and twist-3 distribution amplitudes obtained in the QCD sum rules are collected in section III. Then the sum rules for the various form factors on the light-cone are derived in section IV with the standard correlation function to the leading Fork state. After grouping the input parameters, the numerical computations of form factors in light-cone QCD sum rules are performed in section IV. Subsequently, we apply these form factors to analyze the decay rates of $\bar{B}_0 \to a_0(1450) l \bar{\nu}_l$,

$\bar{B}_0 \to K_0^*(1430) l\bar{l}$, $B_s \to K_0^*(1430) l\bar{\nu}_l$ and $B_s \to f_0(1500) l\bar{l}$ as well as the longitudinal lepton polarization asymmetry for the modes induced by the flavor-changing neutral current, where a brief analysis on comparisons with the results obtained in the light-front quark model and QCD sum rules are also included in this section. The last section is devoted to the conclusions.

II. EFFECTIVE HAMILTONIAN AND PARAMETERIZATIONS OF MATRIX ELEMENT

A. Effective Hamiltonian for $b \to u, s$ transition

Integrating out the particles including top quark, $W^{\pm}$ and Z bosons above scale $\mu = O(m_b)$, we arrive at the effective Hamiltonian responsible for the $b \to u$ transition

$$\mathcal{H}_{eff}(b \to u l \bar{\nu}_l) = \frac{G_F}{\sqrt{2}} V_{ub} \bar{u} \gamma_\mu (1-\gamma_5) b\, \bar{l} \gamma^\mu (1-\gamma_5) \nu_l + h.c.\,, \tag{1}$$

where V_{ub} is the corresponding Cabbibo-Kobayashi-Maskawa (CKM) matrix element and $l = (e, \mu, \tau)$.

Similarly, the effective Hamiltonian revelent to the flavor-changing neutral current (FCNC) transition $b \to s$ can be derived as

$$\begin{aligned}\mathcal{H}_{eff}(b \ \to \ s l\bar{l}) = & \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* [C_9^{eff}(\mu)\, \bar{s}\gamma_\mu (1-\gamma_5) b\, \bar{l}\gamma^\mu(1-\gamma_5) l + C_{10} \bar{s}\gamma_\mu(1-\gamma_5) b\, \bar{l}\gamma^\mu \gamma_5) l \\ & - \frac{2 m_b C_7(\mu)}{q^2} \sigma_{\mu\nu}(1-\gamma_5) q^\nu b\, \bar{l}\gamma^\mu l] + h.c.\,,\end{aligned} \tag{2}$$

where we have neglected the terms proportional to $V_{ub}V_{us}^*$ on account of $|V_{ub}V_{us}^*/V_{tb}V_{ts}^*| < 0.02$. The C_i involved in Eq. (2) are the Wilson coefficients and their particular expressions are given in Ref. [54]. We should emphasize that the Wilson coefficient C_{10} does not renormalize under QCD corrections and hence is independent on the energy scale $\mu \simeq O(m_b)$, since the operator $O_{10} = \bar{s}\gamma_\mu(1-\gamma_5) b \bar{l}\gamma^\mu\gamma_5 l$ can not be induced by the insertion of four-quark operators due to the absence of Z boson in the effective theory. Moreover, the above quark decay amplitude can also receive additional contributions from the matrix element of four-quark operators, which are usually absorbed into the effective Wilson coefficient $C_9^{eff}(\mu)$. To be more specific, we can decompose $C_9^{eff}(\mu)$ into the following three parts [55–61]

$$C_9^{eff}(\mu) = C_9(\mu) + Y_{SD}(z, s') + Y_{LD}(z, s'), \tag{3}$$

where the parameters z and s' are defined as $z = m_c/m_b,\ s' = q^2/m_b^2$. $Y_{SD}(z, s')$ describes the short-distance contributions from four-quark operators far away form the $c\bar{c}$ resonance regions, which can be calculated reliably in perturbative theory. The long-distance contributions $Y_{LD}(z, s')$ from four-quark operators near the $c\bar{c}$ resonance cannot be calculated from first principles of QCD and are usually parameterized in the form of a phenomenological Breit-Wigner formula, which will be neglected in this work due to the absence of experimental data on $B_{(s)} \to J/\psi S$.

The manifest expressions for $Y_{SD}(z, s')$ can be written as [54]

$$\begin{aligned} Y_{SD}(z, s') \;=\; & h(z, s')(3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)) \\ & - \frac{1}{2} h(1, s')(4C_3(\mu) + 4C_4(\mu) + 3C_5(\mu) + C_6(\mu)) \\ & - \frac{1}{2} h(0, s')(C_3(\mu) + 3C_4(\mu)) + \frac{2}{9}(3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)), \end{aligned} \tag{4}$$

with

$$
\begin{aligned}
h(z,s') &= -\frac{8}{9}\ln z+\frac{8}{27}+\frac{4}{9}x-\frac{2}{9}(2+x)|1-x|^{1/2}\begin{cases}\ln\left|\frac{\sqrt{1-x}+1}{\sqrt{1-x}-1}\right|-i\pi & \text{for } x\equiv 4z^2/s'<1\\ 2\arctan\frac{1}{\sqrt{x-1}} & \text{for } x\equiv 4z^2/s'>1\end{cases},\\
h(0,s') &= \frac{8}{27}-\frac{8}{9}\ln\frac{m_b}{\mu}-\frac{4}{9}\ln s'+\frac{4}{9}i\pi\ .
\end{aligned}
\tag{5}
$$

Besides, the non-factorizable effects from the charm quark loop can bring about further corrections to the radiative $b\to s\gamma$ transition, which can be absorbed into the effective Wilson coefficient C_7^{eff} as usual [62–65]. Specifically, the Wilson coefficient C_7^{eff} is given by [66]

$$
C_7^{eff}(\mu)=C_7(\mu)+C'_{b\to s\gamma}(\mu),
\tag{6}
$$

with

$$
C'_{b\to s\gamma}(\mu) = i\alpha_s[\frac{2}{9}\eta^{14/23}(G_1(x_t)-0.1687)-0.03C_2(\mu)],
\tag{7}
$$

$$
G_1(x) = \frac{x(x^2-5x-2)}{8(x-1)^3}+\frac{3x^2\ln^2 x}{4(x-1)^4},
\tag{8}
$$

where $\eta=\alpha_s(m_W)/\alpha_s(\mu)$, $x_t=m_t^2/m_W^2$, $C'_{b\to s\gamma}$ is the absorptive part for the $b\to sc\bar{c}\to s\gamma$ rescattering and we have dropped out the tiny contributions proportion to CKM sector $V_{ub}V_{us}^*$.

B. Parameterizations of hadronic matrix element

With the free quark decay amplitude available, we can proceed to calculate the decay amplitudes for semi-leptonic decays of $B_{q'}\to S$ at hadronic level, which can be obtained by sandwiching the free quark amplitudes between the initial and final meson states. Consequently, the following two hadronic matrix elements

$$
\langle S(p)|\bar{s}\gamma_\mu\gamma_5 b|B_{q'}(p+q)\rangle,\ \ \langle S(p)|\bar{s}\sigma_{\mu\nu}\gamma_5 q^\nu b|B_{q'}(p+q)\rangle
\tag{9}
$$

need to be computed as can be observed from Eqs. (1) and (2). The contributions from vector and tensor types of transitions vanish due to parity conservations which is the property of strong interactions. Generally, the above two matrix elements can be parameterized in terms of a series of form factors as

$$
\langle S(p)|\bar{s}\gamma_\mu\gamma_5 b|B_{q'}(p+q)\rangle = -i[f_+(q^2)p_\mu+f_-(q^2)q_\mu],
\tag{10}
$$

$$
\langle S(p)|\bar{s}\sigma_{\mu\nu}\gamma_5 q^\nu b|B_{q'}(p+q)\rangle = -\frac{1}{m_B+m_S}\left[(2p+q)_\mu q^2-\left(m_B^2-m_S^2\right)q_\mu\right]f_T\left(q^2\right).
\tag{11}
$$

Utilizing the covariant trace formalism introduced in [67], the form factors at large recoil should satisfy the following relations

$$
f_+(q^2)=\frac{2m_B}{m_B+m_S}f_T(q^2),\qquad f_-(q^2)=0,
\tag{12}
$$

where the corrections due to hard gluon exchange are neglected [68]. We can also derive the relation [1] between $f_-(q^2)$ and $f^T(q^2)$ as

$$
f_T(q^2)=-\frac{m_b-m_{q_2}}{m_B-m_S}f_+(q^2),
\tag{13}
$$

---

[1] This relation has been derived in Ref. [69], where the convention $\sigma_{\mu\nu}=-\frac{i}{2}(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)$ is adopted . Therefore, the relations between $f_-(q^2)$ and $f_T(q^2)$ there differ from that given in this paper with a minus sign.

with the help of the equation of motion in the heavy quark limit [70].

III. DISTRIBUTION AMPLITUDES OF SCALAR MESONS

In this section, we would like to collect some revelent information on the distribution amplitudes for scalar mesons, which are the essential ingredients in the sum rules for the $B \to S$ transition form factors.

Up to the leading Fock states, the light-cone distributions of scalar mesons made up of $q_2\bar{q}_1$ can be defined as [3, 53]:

$$\begin{aligned}
\langle S(p)|\bar{q}_2(x)\gamma_\mu q_1(y)|0\rangle &= p_u \int_0^1 du e^{i(up\cdot x+\bar{u}p.y)}\Phi_S(u,\mu), \\
\langle S(p)|\bar{q}_2(x) q_1(y)|0\rangle &= m_S \int_0^1 du e^{i(up\cdot x+\bar{u}p.y)}\Phi_S^s(u,\mu), \\
\langle S(p)|\bar{q}_2(x)\sigma_{\mu\nu} q_1(y)|0\rangle &= -m_S(p_\mu z_\nu - p_\nu z_\mu)\int_0^1 du e^{i(up\cdot x+\bar{u}p.y)}\Phi_S^\sigma(u,\mu),
\end{aligned} \tag{14}$$

where $z = x - y$, m_S is the mass of corresponding scalar meson, $\bar{u} = 1 - u$ and u is the momentum fraction carried by the quark q_2 in the scalar meson. Here $\Phi_S(u,\mu)$ is of twist-2, $\Phi_S^s(u,\mu)$ and $\Phi_S^\sigma(u,\mu)$ are of twist-3. $\Phi_S(u,\mu)$ and ($\Phi_S^s(u,\mu)$, $\Phi_S^\sigma(u,\mu)$) are anti-symmetric and symmetric under the replacement of $u \to 1-u$ in the SU(3) limit owing to the conservation of G parity. To be more specific, the normalizations of $\Phi_S(u,\mu)$, $\Phi_S^s(u,\mu)$ and $\Phi_S^\sigma(u,\mu)$ are given by

$$\int_0^1 du\Phi_S(u,\mu) = f_S, \quad \int_0^1 du\Phi_S^s(u,\mu) = \int_0^1 du\Phi_S^\sigma(u,\mu) = \bar{f}_S. \tag{15}$$

The vector current decay constant f_S defined by

$$\langle S(p)|\bar{q}_2\gamma^\mu q_1|0\rangle = f_S p^\mu \tag{16}$$

should vanish in the SU(3) limit and can be related to the scalar density decay constant $\bar{f}_S$ determined by

$$\langle S(p)|\bar{q}_2 q_1|0\rangle = m_S\bar{f}_S \tag{17}$$

with the help of the equations of motion

$$\bar{f}_S = \mu_S f_S, \qquad \mu_S = \frac{m_S}{m_2(\mu) - m_1(\mu)}, \tag{18}$$

with m_1 and m_2 being the masses of quarks q_1 and q_2 respectively. It should be emphasized that scalar density meson decay constant $\bar{f}_S$ depends on the renormalization scale μ, whereas the the vector current decay constant f_S does not renormalize under the QCD corrections due to the conservation of vector current.

In view of the conformal symmetry hidden in the QCD Lagrangian, the distribution amplitudes of scalar mesons $\Phi_S(u,\mu)$, $\Phi_S^s(u,\mu)$ and $\Phi_S^\sigma(u,\mu)$ can be expanded in the Hilbert space by Jacobbi polynomials with increasing conformal spin as

$$\begin{aligned}
\Phi_S(u,\mu) &= \bar{f}_S(\mu)6u\bar{u}\left[B_0(\mu) + \sum_{m=1}^{\infty} B_m(\mu)C_m^{3/2}(2u-1)\right], \\
\Phi_S^s(u,\mu) &= \bar{f}_S(\mu)\left[1 + \sum_{m=1}^{\infty} a_m(\mu)C_m^{1/2}(2u-1)\right], \\
\Phi_S^\sigma(u,\mu) &= \bar{f}_S(\mu)6u\bar{u}\left[1 + \sum_{m=1}^{\infty} b_m(\mu)C_m^{3/2}(2u-1)\right],
\end{aligned} \tag{19}$$

TABLE I: Decay constants and Gegenbauer moments for the twist-2 distribution amplitude Φ_S of scalar mesons at the scale $\mu = 1$GeV [3].

state	$\bar{f}$(MeV)	B_1	B_3
$a_0(1450)$	460 ± 50	-0.58 ± 0.12	-0.49 ± 0.15
$K_0^*(1430)$	445 ± 50	-0.57 ± 0.13	-0.42 ± 0.22
$f_0(1500)$	490 ± 50	-0.48 ± 0.11	-0.37 ± 0.20

TABLE II: Gegenbauer moments for the twist-3 distribution amplitudes Φ_S^s and Φ_S^σ of scalar mesons at the scale $\mu = 1$GeV [53].

state	$a_1(\times 10^{-2})$	a_2	a_4	$b_1(\times 10^{-2})$	b_2	b_4
a_0	0	$-0.33 \sim -0.18$	$-0.11 \sim 0.39$	0	$0 \sim 0.058$	$0.070 \sim 0.20$
K_0^*	$1.8 \sim 4.2$	$-0.33 \sim -0.025$	—	$3.7 \sim 5.5$	$0 \sim 0.15$	—
f_0	0	$-0.33 \sim -0.18$	$0.28 \sim 0.79$	0	$-0.15 \sim -0.088$	$0.044 \sim 0.16$

where Gegenbauer polynomial $C_m^{3/2}(x)$ can be considered as a special type of Jacobbi polynomials $P_{m+2}^{1,1}(x) \sim C_m^{3/2}(x)$. Combining Eqs.(15), (18) and (19), the zeroth Gegenbauer moment $B_0(\mu)$ for twist-2 distribution amplitude $\Phi_S(u,\mu)$ is given by

$$B_0 = \ \mu_S^{-1}. \tag{20}$$

Moreover, decay constants of scalar mesons and various Gengauber moments B_m, a_m and b_m for both twist-2 and twist-3 LCDAs have been computed in Refs. [3, 53] based on QCD sum rules approach, which are collected in Table I and II.

IV. LIGHT CONE SUM RULES FOR FORM FACTORS

With the LCDAs of scalar mesons available, we are now in a position to derive the sum rules of transition form factors which are responsible for $B_{(s)} \to S$ decays. The basics object in LCSR approach is the correlation function in which one of the hadron is represented by the interpolating current with proper quantum number, such as spin, isospin, (charge) parity and so on; and the other is described by its vector state manifestly. Information on the hadronic transition form factor can be extracted by matching the Green function calculated in two different representations, i.e., phenomenological and theoretical forms, with the help of dispersion relation under the assumption of quark-hadron duality.

A. Light-cone sum rules for the form factors $f_+(q^2)$ and $f_-(q^2)$

Following the standard procedure of sum rules, we consider the correlation function associating with the form factors $f_+\left(q^2\right)$ and $f_-\left(q^2\right)$ determined by the matrix element

$$\Pi_\mu\left(p,q\right) = -\int d^4x e^{iqx}\left\langle S\left(p\right)\left|T\left\{j_{2\mu}(x),j_1(0)\right\}\right|0\right\rangle, \tag{21}$$

where the current $j_{2\mu}(x)=\bar{q}_2\left(x\right)\gamma_\mu\gamma_5 b\left(x\right)$ describes the weak transition of b to q_2 and $j_1(0)=b\left(0\right)i\gamma_5 q_1\left(0\right)$ represents the B_{q_1} channel. In addition, the vacuum-to-meson matrix element for the interpolating current can be given by

$$\left\langle B_{q_1}\left|\bar{b}i\gamma_5 q\right|0\right\rangle = \frac{m^2_{B_{q_1}}}{m_b+m_{q_1}}f_{B_{q_1}}. \tag{22}$$

Inserting the complete set of states between the currents in Eq. (21) with the same quantum numbers as B_{q_1}, we can arrive at the hadronic representation of the correlator (21):

$$\begin{aligned}\Pi_\mu\left(p,q\right) &= i\frac{\langle S\left(p\right)\left|\bar{q}_2\left(0\right)\gamma_\mu\gamma_5 b\left(0\right)\right|B_{q_1}(p+q)\rangle\langle B_{q_1}(p+q)|\bar{b}\left(0\right)i\gamma_5 q_1\left(0\right)|0\rangle}{m^2_{B_{q_1}}-(p+q)^2}\\ &+\sum_h i\frac{\langle S\left(p\right)\left|\bar{q}_2\left(0\right)\gamma_\mu\gamma_5 b\left(0\right)\right|h(p+q)\rangle\langle h(p+q)|\bar{b}\left(0\right)i\gamma_5 q_1\left(0\right)|0\rangle}{m^2_h-(p+q)^2},\end{aligned} \tag{23}$$

where we have separated the contributions from the ground state and higher states corresponding to the B_{q_1} meson channel. Combining the Eqs. (10) , (22) and (23), the phenomenological representations of correlation function (21) can be derived as

$$\Pi_\mu\left(p,q\right) = \frac{m^2_{B_{q_1}}f_{B_{q_1}}}{\left(m_b+m_{q_1}\right)\left[m^2_{B_{q_1}}-(p+q)^2\right]}\left[f_+(q^2)p_\mu+f_-(q^2)q_\mu\right]+\int_{s_0^{B_{q_1}}}^\infty ds\frac{\rho^h_+(s,q^2)p_\mu+\rho^h_-(s,q^2)q_\mu}{s-(p+q)^2}, \tag{24}$$

where we have expressed the contributions from higher states of the B_{q_1} channel in the form of dispersion integral with $s_0^{B_{q_1}}$ being the threshold parameter corresponding to the B_{q_1} channel.

On the theoretical side, the correlation function (21) can also be computed in the perturbative theory with the help of OPE technique at the deep Euclidean region $p^2,\ q^2=-Q^2\ll 0$:

$$\begin{aligned}\Pi_\mu(p,q) &= \Pi_+^{QCD}(q^2,(p+q)^2)p_\mu+\Pi_-^{QCD}(q^2,(p+q)^2)q_\mu\\ &= \int_{(m_b+m_{q_1})^2}^\infty ds\frac{1}{\pi}\frac{\mathrm{Im}\,\Pi_+^{QCD}(s,q^2)}{s-(p+q)^2}p_\mu+\int_{(m_b+m_{q_1})^2}^\infty ds\frac{1}{\pi}\frac{\mathrm{Im}\,\Pi_-^{QCD}(s,q^2)}{s-(p+q)^2}q_\mu.\end{aligned} \tag{25}$$

Making use of the quark-hadron duality assumption

$$\rho_i^h(s,q^2)=\frac{1}{\pi}\mathrm{Im}\,\Pi_i^{QCD}(s,q^2)\Theta(s-s_0^h), \tag{26}$$

with $i=$ “$+,-$” and performing the Borel transformation

$$\hat{\mathcal{B}}_{M^2}=\lim_{\substack{-(p+q)^2,n\to\infty\\ -(p+q)^2/n=M^2}}\frac{(-(p+q)^2)^{(n+1)}}{n!}\left(\frac{d}{d(p+q)^2}\right)^n, \tag{27}$$

with variable $(p+q)^2$ to both two representations of the correlation function, we can finally derive the sum rules for the form factors

$$f_i(q^2)=\frac{m_b+m_{q_1}}{\pi f_{B_{q_1}}m^2_{B_{q_1}}}\int_{(m_b+m_{q_1})^2}^{s_0^{B_{q_1}}}\mathrm{Im}\,\Pi_i^{QCD}(s,q^2)\exp\left(\frac{m^2_{B_{q_1}}-s}{M^2}\right)ds. \tag{28}$$

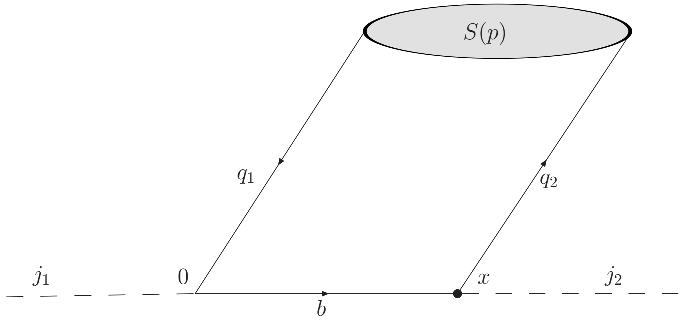

FIG. 1: The tree level contributions to the correlation function Eq. (21), where the current $j_1(0)$ represents the B_{q_1} channel and the current $j_2(x)$ describes the $b \to q_2$ transition.

To the leading order of α_s, the correlation function can be calculated by contracting the bottom quark field in Eq. (21) and inserting the free b quark propagator

$$\Pi_\mu(p,q) = i\int d^4x \int \frac{d^4k}{(2\pi)^4}\frac{e^{i(q-k)x}}{m_b^2-k^2}\langle S(p)|\bar{q}_2(x)\gamma_\mu\gamma_5(\not{k}+m_b)i\gamma_5 q_1(0)|0\rangle, \tag{29}$$

which can be represented by Fig. (1) intuitively. It should be pointed out that the full quark propagator also receives corrections from the background field [71, 72] and can be written as

$$\langle 0|T\{b_i(x)\bar{b}_j(0)\}|0\rangle = \delta_{ij}\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\frac{i}{\not{k}-m_b} - ig\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\int_0^1 dv[\frac{1}{2}\frac{\not{k}+m_b}{(m_b^2-k^2)^2}G_{ij}^{\mu\nu}(vx)\sigma_{\mu\nu} + \frac{1}{m_b^2-k^2}vx_\mu G^{\mu\nu}(vx)\gamma_\nu], \tag{30}$$

where the first term is the free-quark propagator and $G_{ij}^{\mu\nu} = G_{\mu\nu}^a T_{ij}^a$ with $\mathrm{Tr}[T^aT^b] = \frac{1}{2}\delta^{ab}$. Substituting the second term proportional to the gluon field strength into the correlation function can result in the distribution amplitudes corresponding to the higher Fock states of scalar mesons. It is expected that such corrections associating with the LCDAs of higher Fock states do not play any significant role in the sum rules for transition form factors [73], and so can be safely neglected.

Substituting Eq. (14) into Eq. (29) and performing the the integral in the coordinate space, we can achieve the correlation function in the momentum representation at the quark level as

$$\begin{aligned}\Pi_\mu(p,q) &= p_\mu\int_0^1 du\frac{1}{m_b^2-(q+up)^2}\left\{-m_b\Phi_S(u) + um_S\Phi_S^s(u) + \frac{1}{6}m_S\Phi_S^\sigma(u)\left[2+\frac{m_b^2-u^2p^2+q^2}{m_b^2-(q+up)^2}\right]\right\}\\ &\quad +q_\mu\int_0^1 du\frac{1}{m_b^2-(q+up)^2}\left\{m_S\Phi_S^s(u) + \frac{m_S}{6u}\Phi_S^\sigma(u)\left[1-\frac{m_b^2+u^2p^2-q^2}{m_b^2-(q+up)^2}\right]\right\}\\ &\equiv \Pi_+^{QCD}(q^2,(p+q)^2)p_\mu + \Pi_-^{QCD}(q^2,(p+q)^2)q_\mu.\end{aligned} \tag{31}$$

Combining the Eqs. (28) and (31), we can finally arrive at the sum rules for form factors $f_+(q^2)$ and $f_-(q^2)$ as

below

$$f_+\left(q^2\right) = \frac{(m_b+m_{q_1})}{m_{B_{q_1}}^2 f_{B_{q_1}}}\exp\left(\frac{m_B^2}{M^2}\right)\left\{\int_{u_0}^1\frac{du}{u}\exp\left[-\frac{m_b^2+u\bar{u}p^2-\bar{u}q^2}{uM^2}\right]\times\left[\left(-m_b\Phi_S(u)+m_S\left(u\Phi_S^s(u)+\frac{1}{3}\Phi_S^\sigma(u)\right)\right)\right.\right.$$
$$\left.\left.+\frac{1}{uM^2}\frac{m_S}{6}\Phi_S^\sigma(u)\left(m_b^2+u^2p^2+q^2\right)\right]+\frac{m_S}{6}\Phi_S^\sigma(u_0)\exp\left(-\frac{s_0}{M^2}\right)\frac{m_b^2-u_0p^2+q^2}{m_b^2+u_0^2p^2-q^2}\right\}, \quad (32)$$

$$f_-\left(q^2\right) = \frac{(m_b+m_{q_1})}{m_{B_{q_1}}^2 f_{B_{q_1}}}\exp\left(\frac{m_B^2}{M^2}\right)\left\{\int_{u_0}^1\frac{du}{u}\exp\left[-\frac{m_b^2+u\bar{u}p^2-\bar{u}q^2}{uM^2}\right]\times\left[\left(m_S\left(\Phi_S^s(u)+\frac{1}{6u}\Phi_S^\sigma(u)\right)\right)\right.\right.$$
$$\left.\left.-\frac{1}{u^2M^2}\frac{m_S}{6}\Phi_S^\sigma(u)\left(m_b^2+u^2p^2-q^2\right)\right]-\frac{m_S}{6u_0}\Phi_S^\sigma(u_0)\exp\left(-\frac{s_0}{M^2}\right)\right\}, \quad (33)$$

with

$$u_0 = \frac{-(s_0-q^2-p^2)+\sqrt{(s_0-q^2-p^2)^2+4p^2(m_b^2-q^2)}}{2p^2}. \quad (34)$$

As can be observed from the sum rules (32) and (33), both twist-2 and twist-2 distribution amplitudes of scalar mesons can contribute to the form factor $f_+(q^2)$, whereas the other one $f_-(q^2)$ can only receive the contributions form twist-3 LCDAs and should be heavily suppressed in the large recoil region, which is also in agreement with the relations (12) presented in [68].

B. Light-cone sum rules for the form factor $f_T(q^2)$

As for the form factor $f_T\left(q^2\right)$ involved in $b\to s$ transition, we start with the following correlation function

$$\tilde{\Pi}_\mu(p,q) = -\int d^4x e^{iqx}\left\langle S(p)\left|T\left\{\tilde{j}_{2\mu}(x),j_1(0)\right\}\right|0\right\rangle \quad (35)$$

where the current $\tilde{j}_{2\mu}(x)$ is given by

$$\tilde{j}_{2\mu}(x) = \bar{q}_2(x)\sigma_{\mu\nu}a^\nu\gamma_5 b(x)\ . \quad (36)$$

One can write the phenomenological representation of the correlation function at the hadronic level simply by repeating the procedure given above as

$$\tilde{\Pi}_\mu(p,q) = \frac{m_{B_{q_1}}^2 f_{B_{q_1}}}{(m_b+m_{q_1})\,[m_{B_{q_1}}^2-(p+q)^2](m_B+m_S)}[(2p+q)_\mu q^2-q_\mu(m_B^2-m_S^2)]f_T(q^2)$$
$$+\int_{s_0^{B_{q_1}}}^\infty ds\frac{1}{s-(p+q)^2}[-p_\mu q^2+q_\mu(p\cdot q)]\rho_T^h(s,q^2). \quad (37)$$

On the other hand, the correlation function at the quark level can be calculated in the framework of perturbative theory to the leading order of α_s as

$$\tilde{\Pi}_\mu(p,q) = [-p_\mu q^2+q_\mu(p\cdot q)]\int_0^1 du\frac{1}{m_b^2-(q+up)^2}\left\{\Phi_S(u)-\frac{m_b m_S}{3}\frac{\Phi_S^\sigma(u)}{m_b^2-(q+up)^2}\right\}. \quad (38)$$

Matching the correlation function obtained in the two different representations and performing the Borel transformation with respect to the variable $(p+q)^2$, we can achieve the sum rules for the form factor $f_T(q^2)$

$$f_T\left(q^2\right) = \frac{(m_b+m_{q_1})(m_B+m_S)}{m_{B_{q_1}}^2 f_{B_{q_1}}}\exp\left(\frac{m_B^2}{M^2}\right)\left\{-\frac{1}{2}\int_{u_0}^1\frac{du}{u}\exp\left[-\frac{m_b^2+u\bar{u}p^2-\bar{u}q^2}{uT}\right]\times\left[\Phi_S(u)-\frac{m_B m_S}{3uM^2}\Phi_S^\sigma(u)\right]\right.$$
$$\left.+\frac{m_b m_S}{6}\Phi_S^\sigma(u_0)\exp\left(-\frac{s_0}{M^2}\right)\frac{1}{m_b^2+u_0^2p^2-q^2}\right\}. \quad (39)$$

V. NUMERICAL ANALYSIS OF TRANSITION FORM FACTORS

Now we are going to analyze the sum rules for the form factors numerically. Firstly, we collect the input parameters used in this paper as below [74–81]:

$$
\begin{aligned}
&G_F = 1.166 \times 10^{-2}\mathrm{GeV}^{-2}, && |V_{ub}| = 3.96^{+0.09}_{-0.09} \times 10^{-3}, \\
&|V_{tb}| = 0.9991, && |V_{ts}| = 41.61^{+0.10}_{-0.80} \times 10^{-3}, \\
&m_b = (4.68 \pm 0.03)\mathrm{GeV}, && m_s(1\mathrm{GeV}) = 142\mathrm{MeV}, \\
&m_u(1\mathrm{GeV}) = 2.8\mathrm{MeV}, && m_d(1\mathrm{GeV}) = 6.8\mathrm{MeV}, \\
&m_{B_0} = 5.279\mathrm{GeV}, && m_{B_s} = 5.368\mathrm{GeV}, \\
&f_{B_0} = (0.19 \pm 0.02)\mathrm{GeV}, && f_{B_s} = (0.23 \pm 0.02)\mathrm{GeV}.
\end{aligned} \tag{40}
$$

It is noted that the input values for f_B and f_{B_s} are in agreement with the unquenched lattice results [79] $f_B = 0.216 \pm\ 0.022$ GeV and $f_{B_s} = 0.259 \pm\ 0.032$ GeV, and with the results from the QCD sum rules [80, 81]. The threshold parameter s can be determined by the condition that the sum rules should take on the best stability in the allowed Borel region. Besides, the values of threshold parameter should be around the mass square of the corresponding first excited state, hence they are also chosen the same as that in the usual two-point QCD sum rules. The standard value of the threshold in the X channel is $s_{0X} = (m_X + \Delta_X)^2$, where Δ_X is approximately taken to be 0.6GeV in the literature [82–85] . To be more specific, we adopt the threshold parameters $s_0^{B_0} = (35 \pm 2)\mathrm{GeV}^2$ and $s_0^{B_s} = (36 \pm 2)\mathrm{GeV}^2$ corresponding to B_0 and B_s channels respectively, for the error estimate in the numerical analysis.

With all the parameters, we can proceed to compute the numerical values of the form factors. In principle, the form factors $f_+(q^2)$, $f_-(q^2)$ and $f_T(q^2)$ should not depend on the Borel mass M^2 in a complete theory. However, as we truncate the operator product expansion up to the leading conformal spin of distribution amplitudes for scalar mesons in the leading Fock configuration and keep the perturbative expansion in α_s to leading order, a manifest dependence of the form factors on the Borel parameter M^2 would emerge in practice. Therefore, one should look for a working "window", where the results only mildly vary with respect to the Borel mass, to make the truncation reasonable and acceptable.

Firstly, we concentrate on the form factors at zero momentum transfer. As for the form factors $f_+(0)$ involved in $\bar{B}_0 \to a_0(1450) l \nu_l$, we require that the contributions from the higher excited resonances and continuum states hold the fraction less than 20 % in the total sum rules and the value of $f_+(0)$ does not vary drastically within the selected region for the Borel mass. In view of these considerations, the Borel parameter M^2 should not be too large in order to ensure that the contributions from the higher states are exponentially damped as can be observed from Eqs. (32), (33) and (39) and the global quark-hadron duality is satisfactory; on the other hand, the number of Borel mass also could not be too small for the sake of validity of OPE near the light-cone for the correlation function in deep Euclidean region, since the contributions of higher twist distribution amplitudes amount the higher order of $1/M^2$ to the perturbative part. Subsequently, we indeed find the Borel platform $M^2 \in [10, 15]\mathrm{GeV}^2$ with the selected threshold parameter $s_0^{B_0} = 35\mathrm{GeV}^2$ as shown in Fig. 2, where the ratio of twist-3 distribution amplitudes in the total sum rules are also included for a comparison. Following the same methods, we can also further evaluate all the form factors $f_+(0)$, $f_-(0)$ and $f_T(0)$ associating with the decay modes $\bar{B}_0 \to a_0(1450) l \nu_l$, $\bar{B}_0 \to K_0^*(1430) l \bar{l}$, $B_s \to K_0^*(1430) l \nu_l$, and $B_s \to f_0(1500) l \bar{l}$, whose results have been collected in Table III-VI, where we have combined the uncertainties

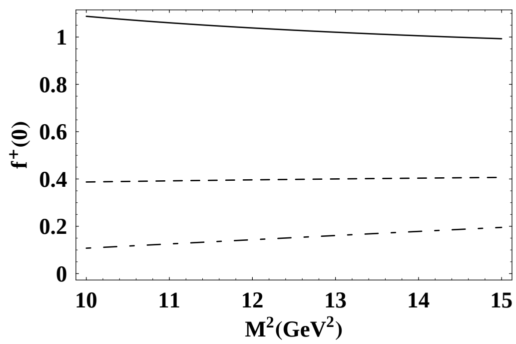

FIG. 2: The solid line denotes dependence of form factors f_+ at $q^2 = 0$ responsible for the decay of $\bar{B}_0 \to a_0(1450) l \nu_l$ on the Borel window $M_B^2 \in [10.0, 15.0]$ GeV^2 with the chosen threshold parameter $s_0^{B_{q_1}} = 35\text{GeV}^2$. The dashed and the dot-dashed lines are the ratio of contribution from twist-3 distribution amplitudes and higher states of B_{q_1} channel in the total sum rules respectively.

from the variation of Borel parameters, fluctuation of threshold value, errors of b quark mass, corrections from decay constants of the involved mesons as well as uncertainties from the Gengenbauer moments in the distribution amplitudes of scalar mesons. It can be observed that the errors on the form factors are estimated within the level of 20 % as expected by the general understanding of the theoretical framework.

Next, we can further investigate the q^2 dependence of the form factors $f_+(q^2)$, $f_-(q^2)$ and $f_T(q^2)$ based on the sum rules given in Eqs. (32), (33) and (39). One usually parameterize the form factors $f_i(q^2)(i = +, -, T)$ in either the single pole form

$$f_i(q^2) = \frac{f_i(0)}{1 - a_i q^2/m_{B_{q_1}}^2}, \tag{41}$$

or the double-pole form

$$f_i(q^2) = \frac{f_i(0)}{1 - a_i q^2/m_{B_{q_1}}^2 + b_i q^4/m_{B_{q_1}}^4}, \tag{42}$$

in the whole kinematical region $0 < q^2 < (m_{B_{q_1}} - m_S)^2$, while non-perturbative parameters a_i and b_i can be fixed by the magnitudes of form factors corresponding to the small and intermediate momentum transfer calculated in the LCSR approach. Our results for the parameters a_i, b_i accounting for the q^2 dependence of form factors f_+, f_- and f_T are grouped in Table III-VI, where the values estimated in other works are also given for a comparison.

TABLE III: Numerical results for the parameters $f_i(0)$, a_i and b_i involved in the (single) double-pole fit of form factors (41), (42) responsible for $\bar{B}_0 \to a_0(1450) l \nu_l$ decay up to the twist-3 distribution amplitudes of scalar mesons, where the numbers derived in the covariant light-front quark model [86] are also collected for a comparison.

	$f_i(0)$	a_i	b_i
f_+	$1.04^{+0.20}_{-0.20}$	$0.98^{+0.08}_{-0.08}$	
	0.52 [86]	1.57 [86]	0.70 [86]
f_-	$0.077^{+0.014}_{-0.014}$	$1.52^{+0.07}_{-0.12}$	
f_T	$0.66^{+0.13}_{-0.14}$	$0.88^{+0.10}_{-0.09}$	

TABLE IV: Numerical results for the parameters $f_i(0)$, a_i and b_i involved in the (single) double-pole fit of form factors (41), (42) responsible for $\bar{B}_0 \to K_0^*(1430) l \bar{l}$ decay up to the twist-3 distribution amplitudes of scalar mesons, where the numbers derived in the covariant light-front quark model [87] and QCD sum rules approach [69] are also collected for a comparison.

	$f_i(0)$	a_i	b_i
f_+	$0.97^{+0.20}_{-0.20}$	$0.86^{+0.19}_{-0.18}$	
	0.52 [87]	1.36 [87]	0.86 [87]
	0.62 ± 0.16 [69]	0.81 [69]	-0.21 [69]
f_-	$0.073^{+0.02}_{-0.02}$	$2.50^{+0.44}_{-0.47}$	$1.82^{+0.69}_{-0.76}$
f_T	$0.60^{+0.14}_{-0.13}$	$0.69^{+0.26}_{-0.27}$	
	0.34 [87]	1.64 [87]	1.72 [87]
	0.26 ± 0.07 [69]	0.41 [69]	-0.32 [69]

VI. DECAY RATE AND POLARIZATION ASYMMETRY

With the transition form factors derived, one can proceed to perform the calculations on some interesting observables in phenomenology, such as decay rate, polarization asymmetry. In particular, the forward-backward asymmetry for the decay modes $\bar{B}_0 \to K_0^*(1430) l \bar{l}$ and $B_s \to f_0(1500) l \bar{l}$ is exactly equal to zero in the SM [89, 90] due to the absence of scalar-type coupling between the lepton pair, which serve as a valuable ground to test the SM precisely as well as bound its extensions stringently.

The semi-leptonic decay $\bar{B}_0 \to K_0^*(1430) l \bar{l}$ is induced by flavor-changing neutral current. The differential decay

TABLE V: Numerical results for the parameters $f_i(0)$, a_i and b_i involved in the (single) double-pole fit of form factors (41), (42) responsible for $B_s \to K_0^*(1430) l \nu_l$ decay up to the twist-3 distribution amplitudes of scalar mesons, where the numbers derived in the QCD sum rules approach [88] are also collected for a comparison.

	$f_i(0)$	a_i	b_i
f_+	$0.83^{+0.26}_{-0.13}$	$0.93^{+0.20}_{-0.06}$	
	0.48 ± 0.20 [88]	$1.25^{+0.07}_{-0.06}$ [88]	
f_-	$0.071^{+0.02}_{-0.02}$	$2.46^{+0.36}_{-0.39}$	$1.72^{+0.59}_{-0.64}$
f_T	$0.52^{+0.18}_{-0.08}$	$0.77^{+0.10}_{-0.07}$	

TABLE VI: Numerical results for the parameters $\xi_i(0)$, a_i and b_i involved in the (single) double-pole fit of form factors (41), (42) responsible for $B_s \to f_0(1500)l\bar{l}$ decay up to the twist-3 distribution amplitudes of scalar mesons.

	$f_i(0)$	a_i	b_i
f_+	$0.86^{+0.15}_{-0.15}$	$1.17^{+0.06}_{-0.05}$	
f_-	$0.056^{+0.015}_{-0.015}$	$1.94^{+0.48}_{-0.85}$	$0.52^{+0.89}_{-1.6}$
f_T	$0.56^{+0.10}_{-0.11}$	$1.09^{+0.08}_{-0.07}$	

width of $\bar{B}_0 \to K_0^*(1430)l\bar{l}$ in the rest frame of $\bar{B}_0$ meson can be written as [74]

$$\frac{d\Gamma(\bar{B}_0 \to K_0^*(1430)l\bar{l})}{dq^2} = \frac{1}{(2\pi)^3}\frac{1}{32m_{\bar{B}_0}}\int_{u_{min}}^{u_{max}} |\widetilde{M}_{\bar{B}_0\to K_0^*(1430)l\bar{l}}|^2 du, \tag{43}$$

where $u = (p_{K_0^*(1430)} + p_l)^2$ and $q^2 = (p_l + p_{\bar{l}})^2$; $p_{K_0^*(1430)}$, p_l and $p_{\bar{l}}$ are the four-momenta vectors of $K_0^*(1430)$, l and $\bar{l}$ respectively; $|\widetilde{M}_{\bar{B}_0\to K_0^*(1430)l\bar{l}}|^2$ is the squared decay amplitude after integrating over the angle between the l and $K_0^*(1430)$ baryon. The upper and lower limits of u are given by

$$\begin{aligned} u_{max} &= (E^*_{K_0^*(1430)} + E^*_l)^2 - \Big(\sqrt{E^{*2}_{K_0^*(1430)} - m^2_{K_0^*(1430)}} - \sqrt{E^{*2}_l - m_l^2}\Big)^2, \\ u_{min} &= (E^*_{K_0^*(1430)} + E^*_l)^2 - \Big(\sqrt{E^{*2}_{K_0^*(1430)} - m^2_{K_0^*(1430)}} + \sqrt{E^{*2}_l - m_l^2}\Big)^2; \end{aligned} \tag{44}$$

where $E^*_{K_0^*(1430)}$ and E^*_l are the energies of $K_0^*(1430)$ and l in the rest frame of lepton pair and can be determined as

$$E^*_{K_0^*(1430)} = \frac{m^2_{\bar{B}_0} - m^2_{K_0^*(1430)} - q^2}{2\sqrt{q^2}}, \qquad E^*_l = \frac{q^2}{2\sqrt{q^2}}. \tag{45}$$

Collecting everything together, we can arrive at the general expression of differential decay rate for $B_{q'} \to Sl\bar{l}$ as [87]:

$$\frac{d\Gamma\left(B_{q'} \to Sl\bar{l}\right)}{ds'} = \frac{G_F^2\left|V_{tb}V_{ts}\right|^2 m_B^5 \alpha_{em}^2}{1536\pi^5}\left(1 - \frac{4r_l}{s'}\right)^{1/2}\varphi_S^{1/2}\left[\left(1 + \frac{2r_l}{s'}\right)\alpha_S + r_l\delta_S\right] \tag{46}$$

where

$$\begin{aligned} s' &= q^2/m_B^2, \qquad r_l = m_l^2/m_B^2, \qquad r_S = m_S^2/m_B^2, \\ \varphi_S &= (1-r_S)^2 - 2s\,(1+r_S) + s^2, \\ \alpha_S &= \varphi_S\left(\left|C_9^{eff}\frac{f_+\left(q^2\right)}{2} - 2\frac{C_7 f_T\left(q^2\right)}{1+\sqrt{r_S}}\right|^2 + \left|C_{10}\frac{f_+\left(q^2\right)}{2}\right|^2\right), \\ \delta_S &= 6\left|C_{10}\right|^2\left\{\left[2\left(1+r_S\right) - s\right]\left|\frac{f_+\left(q^2\right)}{2}\right|^2 + \left(1-r_S\right)\mathrm{Re}\left[f_+\left(q^2\right)\left(f_-(q^2) - \frac{f_+\left(q^2\right)}{2}\right)^*\right] + s\left|f_-(q^2) - \frac{f_+\left(q^2\right)}{2}\right|^2\right\}. \end{aligned}$$

The invariant dilepton mass distribution for $\bar{B}_0 \to K_0^*(1430)l\bar{l}$ as the functions of squared momentum transfer q^2 are presented in Fig. 3. In the same way, we can also estimate the decay rates of $\bar{B}_0 \to K_0^*(1430)l\bar{l}$, $B_s \to K_0^*(1430)l\bar{\nu}_l$ and $B_s \to f_0(1500)l\bar{l}$ with $l = e, \mu, \tau$ based on the form factors calculated in light-cone sum rules. The results of the total decay width corresponding to these decay modes are grouped in Table VII, where the results obtained in other frameworks are also presented for a comparison. As can be observed, the decay rates of the electron- and muon- pair final states are practically the same, while the decay rate of tauon-pair channel is much smaller due to the heavily suppressed phase space.

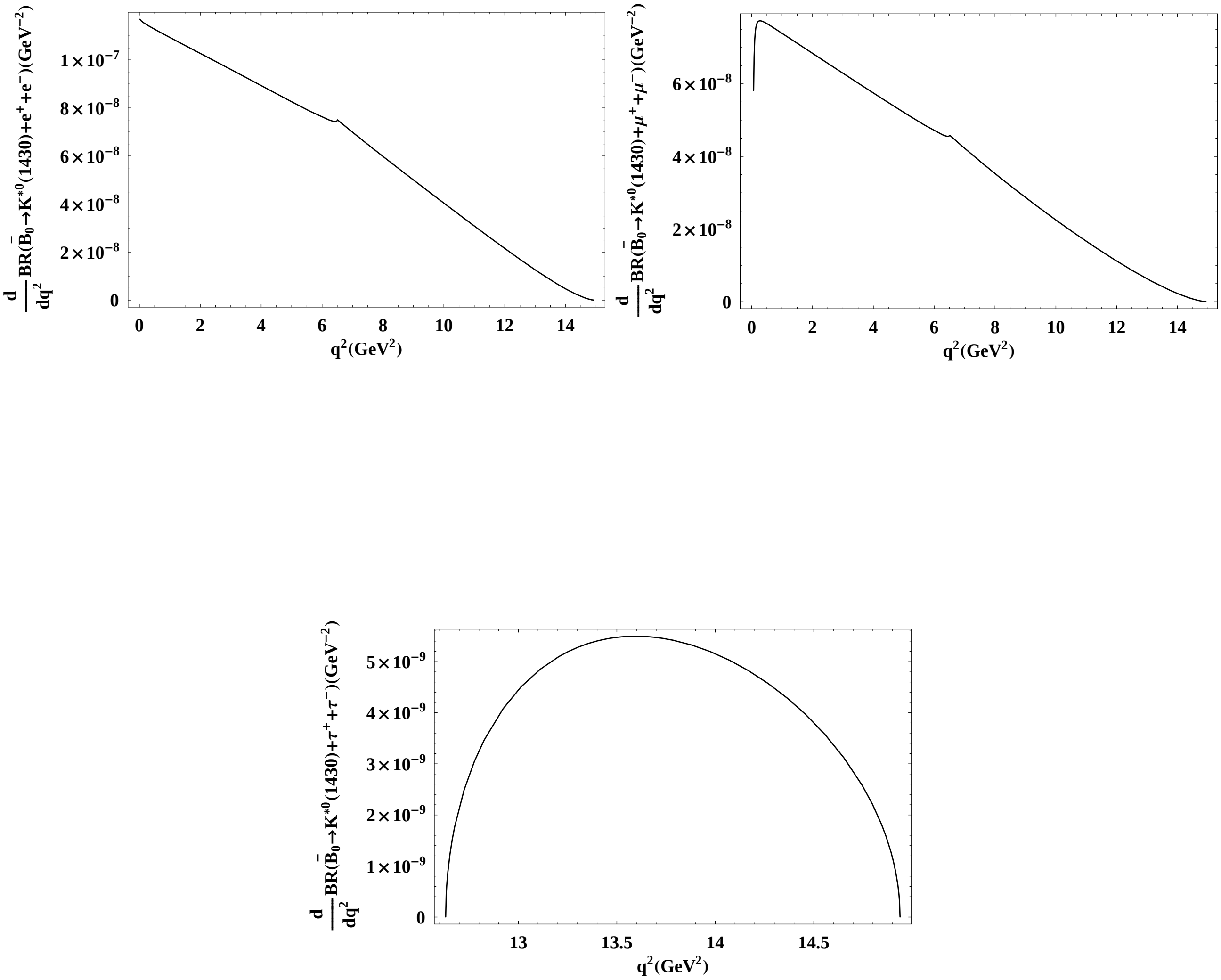

FIG. 3: The invariant dilepton mass distributions for $\bar{B}_0 \to K_0^*(1430)e^+e^-$, $\bar{B}_0 \to K_0^*(1430)\mu^+\mu^-$ and $\bar{B}_0 \to K_0^*(1430)\tau^+\tau^-$ as functions of squared momentum transfer q^2 based on light-cone QCD sum rules.

Another interesting observable in the decay of $\bar{B}_0 \to K_0^*(1430)l\bar{l}$ is the polarization asymmetry of the final state charged leptons, which is very helpful to extract the information on the spin of them. The four-spin vector s^μ of a lepton can be defined in its rest frame as

$$(s^\mu)_{r.s.} = (0,\ \hat{\xi}). \tag{47}$$

TABLE VII: Numerical results for the total decay width of $\bar{B}_0 \to a_0(1450)l\bar{\nu}_l$, $\bar{B}_0 \to K_0^*(1430)l\bar{l}$, $B_s \to K_0^*(1430)l\bar{\nu}_l$ and $B_s \to f_0(1500)l\bar{l}$ with $l = e, \mu, \tau$ in the light-cone sum rules approach, together with the numbers estimated in QCD sum rules [69, 88] and light-front quark model [87].

	$\bar{B}_0 \to a_0(1450)e\bar{\nu}_e$	$\bar{B}_0 \to K_0^*(1430)e^+e^-$	$B_s \to K_0^*(1430)e\bar{\nu}_e$	$B_s \to f_0(1500)e^+e^-$
LCSR	$1.8^{+0.9}_{-0.6} \times 10^{-4}$	$5.7^{+3.4}_{-2.4} \times 10^{-7}$	$1.3^{+1.3}_{-0.4} \times 10^{-4}$	$5.3^{+2.3}_{-1.8} \times 10^{-7}$
LFQM		1.63×10^{-7} [87]		
QCDSR		$(2.09 \sim 2.68) \times 10^{-7}$[69]	$3.6^{+3.8}_{-2.4} \times 10^{-5}$ [88]	
	$\bar{B}_0 \to a_0(1450)\mu\bar{\nu}_\mu$	$\bar{B}_0 \to K_0^*(1430)\mu^+\mu^-$	$B_s \to K_0^*(1430)\mu\bar{\nu}_\mu$	$B_s \to f_0(1500)\mu^+\mu^-$
LCSR	$1.8^{+0.9}_{-0.7} \times 10^{-4}$	$5.6^{+3.1}_{-2.3} \times 10^{-7}$	$1.3^{+1.2}_{-0.4} \times 10^{-4}$	$5.2^{+2.3}_{-1.7} \times 10^{-7}$
LFQM		1.62×10^{-7} [87]		
QCDSR		$(2.07 \sim 2.66) \times 10^{-7}$[69]		
	$\bar{B}_0 \to a_0(1450)\tau\bar{\nu}_\tau$	$\bar{B}_0 \to K_0^*(1430)\tau^+\tau^-$	$B_s \to K_0^*(1430)\tau\bar{\nu}_\tau$	$B_s \to f_0(1500)\tau^+\tau^-$
LCSR	$6.3^{+3.4}_{-2.5} \times 10^{-5}$	$9.8^{+12.4}_{-5.5} \times 10^{-9}$	$5.2^{+5.7}_{-1.8} \times 10^{-5}$	$1.2^{+0.8}_{-0.5} \times 10^{-8}$
LFQM		2.86×10^{-9} [87]		
QCDSR		$(1.70 \sim 2.20) \times 10^{-9}$[69]		

The unit vector along the longitudinal direction of the lepton polarization is given by

$$\hat{e}_L = \frac{\mathbf{p}_l}{|\mathbf{p}_l|}. \tag{48}$$

In this work, we mainly concentrate on the longitudinal lepton polarization asymmetry that can be defined as

$$P_L(s') = \frac{\frac{d\Gamma\left(\hat{e}_L\hat{\xi}=1\right)}{ds'} - \frac{d\Gamma\left(\hat{e}_L\hat{\xi}=-1\right)}{ds'}}{\frac{d\Gamma\left(\hat{e}_L\hat{\xi}=1\right)}{ds'} + \frac{d\Gamma\left(\hat{e}_L\hat{\xi}=-1\right)}{ds'}}, \tag{49}$$

which is a parity-odd but CP-even observable similar to the forward-backward asymmetry. The manifest expression for the longitudinal polarization asymmetry P_L in $B_{q'} \to Sl\bar{l}$ is derived as [87]

$$P_L(s') = \frac{2\left(1 - \frac{4r_l}{s'}\right)^{1/2}}{\left(1 + \frac{2r_l}{s'}\right)\alpha_S + r_l\delta_S}\mathrm{Re}\left[\varphi_S\left(C_9^{eff}\frac{f_+\left(q^2\right)}{2} - 2\frac{C_7 f_T\left(q^2\right)}{1+\sqrt{r_S}}\right)\left(C_{10}\frac{f_+\left(q^2\right)}{2}\right)^*\right] \tag{50}$$

It has been shown that this asymmetry is insensitive to the form factors in the massless limit for the lepton and can be approximated by

$$P_L(s') = \frac{2\mathrm{Re}[C_9^{eff}C_{10}^*]}{|C_9^{eff}|^2 + |C_{10}|^2} + O(C_7) \simeq -1, \tag{51}$$

in view of the smallness of Wilson coefficient C_7 compared with C_9^{eff} and C_{10}. The distribution of the longitudinal polarization asymmetry P_L in $\bar{B}_0 \to K_0^*(1430)l\bar{l}$ as a function of q^2 are presented in Fig. 4, from which we indeed find that $P_L(\bar{B}_0 \to K_0^*(1430)e^+e^-)$ and $P_L(\bar{B}_0 \to K_0^*(1430)\mu^+\mu^-)$ are close to -1 except the end points region.

It is also useful to introduce the integrated longitudinal lepton polarization asymmetry $\langle A_{PL}\rangle$ in order to characterize the typical value of longitudinal lepton polarization asymmetry

$$\langle A_{PL}\rangle = \int_{s'_{min}}^{s'_{max}} A_{PL}(s')ds', \tag{52}$$

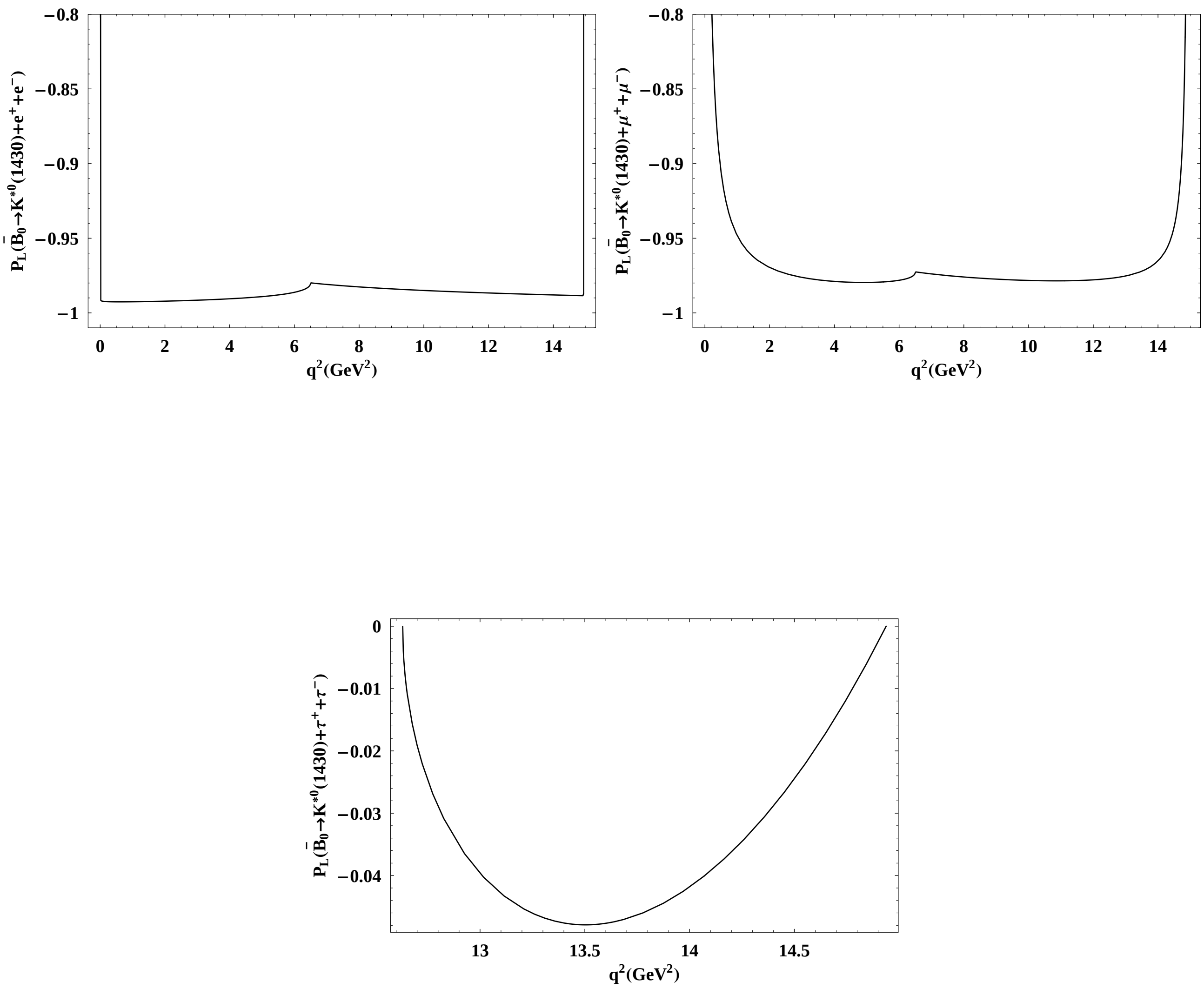

FIG. 4: Lepton polarization asymmetries for $\bar{B}_0 \to K_0^*(1430)e^+e^-$, $\bar{B}_0 \to K_0^*(1430)\mu^+\mu^-$ and $\bar{B}_0 \to K_0^*(1430)\tau^+\tau^-$ as functions of squared momentum transfer q^2 based on light-cone QCD sum rules.

with $s'_{min} = 4m_l^2/m_B^2$ and $s'_{max} = (m_B^2 - m_S^2)/m_B^2$. The numerical results of integrated longitudinal lepton polarization asymmetry have been grouped in Table VIII, together with results in light-front quark model. From this table, we can observe that our results for $\langle A_{PL} \rangle$ are in good agreement with that given by light-front quark model, which also indicates that this asymmetry is not sensitive to the decay form factors.

TABLE VIII: Numerical results of the integrated longitudinal lepton polarization asymmetry for $\bar{B}_0 \to K_0^*(1430) l\bar{l}$ and $B_s \to f_0(1500) l\bar{l}$ with $l = e, \mu, \tau$ in the light-cone sum rules approach, , where the numbers estimated in light-front quark model [87] are also collected here.

	$\bar{B}_0 \to K_0^*(1430) e^+ e^-$	$B_s \to f_0(1500) e^+ e^-$
$\langle A_{PL} \rangle$	-0.99 ± 0.0	-0.99 ± 0.0
	-0.97 [87]	
	$\bar{B}_0 \to K_0^*(1430) \mu^+ \mu^-$	$B_s \to f_0(1500) \mu^+ \mu^-$
$\langle A_{PL} \rangle$	-0.96 ± 0.0	-0.96 ± 0.0
	-0.95 [87]	
	$\bar{B}_0 \to K_0^*(1430) \tau^+ \tau^-$	$B_s \to f_0(1500) \tau^+ \tau^-$
$\langle A_{PL} \rangle$	$-0.03^{+0.00}_{-0.01}$	-0.04 ± 0.0
	-0.03 [87]	

VII. CONCLUSIONS

Within the framework of light-cone sum rules, we analyze the form factors responsible for semi-leptonic decays of $\bar{B}_0 \to a_0(1450) l \bar{\nu}_l$, $\bar{B}_0 \to K_0^*(1430) l\bar{l}$, $B_s \to K_0^*(1430) l \bar{\nu}_l$ and $B_s \to f_0(1500) l\bar{l}$ with $l = e, \mu, \tau$ up to the twist-3 distribution amplitudes for the leading Fock state. Owing to the strong coupling of scalar mesons to the scalar current, the form factors associating with $B \to S$ transition are approximately twice as large as that for the ones in the $B \to P$ case. The form factors $f_+(q^2)$, $f_-(q^2)$ and $f_T(q^2)$ calculated in this work verify the relations derived in the large recoil and heavy quark limit.

Utilizing these form factors, we investigated the branching fractions of $\bar{B}_0 \to a_0(1450) l \bar{\nu}_l$, $\bar{B}_0 \to K_0^*(1430) l\bar{l}$, $B_s \to K_0^*(1430) l \bar{\nu}_l$ and $B_s \to f_0(1500) l\bar{l}$. The magnitudes of $BR(\bar{B}_0 \to a_0(1450) l \bar{\nu}_l)$ and $BR(B_s \to K_0^*(1430) l \bar{\nu}_l)$ can arrive of the order 10^{-4}, while the branching ratios of $\bar{B}_0 \to K_0^*(1430) l\bar{l}$ and $B_s \to f_0(1500) l\bar{l}$ are of the order $10^{-8} \sim 10^{-7}$, which can marginally observed in the future experiments. The longitudinal lepton polarization asymmetries for $\bar{B}_0 \to K_0^*(1430) l\bar{l}$ and $B_s \to f_0(1500) l\bar{l}$ are also considered in the SM. Our results for the asymmetry are in good agreement with that given by the light front quark model. The averaged asymmetries $\langle A_{PL} \rangle$ for the final states including e^+e^- and $\mu^+\mu^-$ are almost equal to -1 except the end points region. However, the tau lepton polarization asymmetries are remarkably small and not measurable due to the efficiency for the detectability of the tauon. The theoretical predictions on the production properties presented in this work are very helpful to clarify the inner structures of scalar mesons as well as understanding the dynamics of strong interactions.

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